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Regional Vector Time Series Approach

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Abstract

Male and female birth of a region can be treated as a vector time series. In this article, we have treated male and female birth of a region of Marathwada of Maharastra state as a vector $\bar{r}_1 = (X_1, X_2, \dots X_5)$ and $r_2 = (Y_1, Y_2, \dots Y_5)$ .. Where X_1 = Male birth at Aurangabad , X_2 = Male birth at Parbhani , X_3 = Male birth at Osmanabad , X_4 = Male birth at Beed and X_5 = Male birth at Nanded and similarly Y_1 = Female birth at Aurangabad , Y_2 = Female birth at Parbhani , Y_3 = Female birth at Osmanabad , Y_4 = Female birth at Beed and Y_5 = Female birth at Nanded. Thus, we get a vector time series, $\bar{T} = (r_{ij})$, i = 1, 2 , …….n years, j = 1,2….5 districts. For two different component vector time series. This opens up very interesting questions. How are the properties of T related to component time series?.

A preliminary discussion of properties of vector time series and possible testing methodology for stationary property precedes the actual application to regional male and female birth data.

Keywords: Time Series, Vector Time Series, Regression Analysis, Auto Covariance, Auto-Correlation.

Introduction

Regional two vector component time series can occur naturally in real life. For example, if we consider the male and female birth over a region, where male and female birth is recorded over a cluster of recording stations, we get a vector time series of male and a vector time series of female birth. To what extent the properties of component time series determine the properties of the regional vector time series is worth looking into.

In what follows are first discussed in relation to, few properties of vector time series and then tried to compute the same for the regional annual male and female birth record of Marathwada by using data from 1970 to 2004.

Objectives of the Study

1. To develop theory of vector time series;
2. To develop algorithms for analyzing vector time series; and
3. To interpret the results of characterization in real social terms.

Basic Concepts

Basic definitions and few properties of vector stationary time series are given in this section.

Definition 2.1: A Random Vector

A random vector, $\bar{X} = (X_1, X_2, \dots X_K)$ is a single valued function whose domain is Ω, whose range is in Euclidean n-space R^n and which is B-measurable, i.e. for every subset R C R^n {ω Є Ω | X_1(ω) … X_K(ω) Є R}Є B. A random vector will also be called an K- dimensional random variable or a vector random variable.

If $X_1, X_2 \dots X_K$ are k random variables and $\bar{X} = (X_1, X_2, \dots X_K)$ is a random vector, [12].

Definition 2.2: A Vector Time Series

Let (Ω, C, P) be a probability space; with Ω sample space; C = σ(Ω). Let T be an index set and N = {1,2... k}. A real valued vector time series is a real valued function $X_{it}(\omega)$, i = 1,2…k defined on N ×T ×Ω such that for each fixed t Є T, i Є N, $X_{it}(\omega)$ is a random variable on (Ω, C, P).

A vector time series can be considered as a collection {X_{it} : t Є T }, i =1,2 ... k of random variables [8].

Definition 2.3: Stationary Vector Time Series

A process whose probability structure does not change with time is called stationary. Broadly speaking a vector time series is said to be stationary, if there is no systematic change in mean i.e. no trend. There is no systematic change in variance.

Let $\bar{X} = (x_1, x_2, \dots x_n)$ be realizations of random variables $(X_1, X_2, \dots X_K)$.

Definition 2.4: Strictly Stationary Vector Time Series

A vector time series is called strictly stationary, if their joint distribution function satisfy

$$F_{X_{1t}}(\bar{x})_{X_{2t}\ldots X_{kt}} = F_{X_{1t+h}}(\bar{x})_{X_{2t+h}\ldots X_{kt+h}} \quad \ldots\ldots\ldots (1)$$

Where, the equality must hold for all possible sets of indices it and (it + h) in the index set. Further the joint distribution depends only on the distance h between the elements in the index set and not on their actual values.

Main Results

Theorm 3.1

If $\{ X_{it} : t \in T \}$, i =1, 2,…k is strictly vector time series with $E\{X_{it}\} < \alpha$ and $E\{X_{it} - \mu\} < \beta$ then ,

$E\{X_{it}\} = E\{X_{it+h}\}$, for all it, h

and $E[\{X_{it} - \mu_i\}\{X_{jt} - \mu_j\}] = E[\{X_{it+h} - \mu_i\}\{X_{jt+h} - \mu_j\}]$, for all it, h } ….(2)

Proof

Proof follows from definition (2.4).

In usual cases above equation (2) is used to determine that a vector time series is stationary.

Definition 3.1: Weakly Stationary Vector Time Series

A vector time series is called weakly stationary if

1. The expected value of X_{it} is a constant for all it.
2. The covariance matrix of $(X_{1t}, X_{2t}, \ldots X_{kt})$ is same as covariance matrix of $(X_{1t+h}, X_{2t+h}, \ldots X_{kt+h})$.

A look in the covariance matrix $(X_{1t} X_{2t} \ldots X_{kt})$ would show that diagonal terms would contain terms covariance (X_{it}, X_{it}) which are essentially variances and off diagonal terms would contains terms like covariance (X_{it}, X_{jt}). Hence, the definitions to follow assume importance. Since these involve elements from the same set $\{X_{it}\}$, the variances and co- variances are called auto-variances and auto-co variances.

Definition 3.2: Auto-Covariance Function

The covariance between $\{X_{it}\}$ and $\{X_{it+h}\}$ separated by h time unit is called auto-covariance at lag h and is denoted by $\Gamma_{ij}(h)$.

$\Gamma_{ij}(h) = \text{cov}(X_{it}, X_{jt+h}) = E\{X_{it} - \mu_i\}\{X_{jt+h} - \mu_j\}$ … .(3)

The matrix $\Gamma\bar{h} = \Gamma_{ij}(h)$ is called the auto covariance matrix function.

Definition 3.3: The Auto Correlation Function

The correlation between observation which are separated by h time unit is called auto-correlation at lag h. It is given by

$$P_{ij}(h) = \frac{E\{X_{it} - \mu_i\}\{X_{jt+h} - \mu_j\}}{[E\{X_{it} - \mu_i\}^2 E\{X_{jt+h} - \mu_j\}^2]^{1/2}} \quad \ldots. (4)$$

$$= \frac{\Gamma_{ij}(h)}{[\Gamma_{ii}(h) \quad \Gamma_{jj}(h)]^{1/2}}$$

Where μ_i is the mean of i^{th} component time series.

Remark 3.1

For a vector stationary time series the variance at time (it+h) is same as that at time it. Thus, the auto correlation at lag h is

$$P_{ij}(h) = \frac{\Gamma_{ij}(h)}{\Gamma_{ii}(0)} \quad \ldots (5)$$

Remark 3.2

For h = 0, we get $\rho_{ij}(0) = 1$.

For application attempts have been made to establish that rainfall at certain districts of Marathwada satisfy equation (1) and (5).

Theorem 3.2

The covariance function of vector stationary time series $\{X_{it} : t \in T\}$ is positive semi-definite function in that

$$\sum_{J=1}^{n}\sum_{k=1}^{n} a_k^T \Gamma(t_j - t_k) a_j \geq 0 ,$$

For any set of real vectors $(a_1, a_2, \ldots a_n)$ and any set of indices $(t_1, t_2 \ldots t_n) \in T$.

Proof

The result can be obtained by evaluating the variance of

$$X = \sum_{j=1}^{n} a_j^T X_{tj.}$$

For this without loss of generality $E(X_{tj}) = 0$. It shows that the variance of a random variable is non-negative i.e. $V(X) \geq 0$.

$$V(X) = V(\Sigma\ a_j^T X_{tj}) \geq 0$$

$$= E\left(\sum_{j=1}^{n} a_j^T X_{tj}^T\right)\left(\sum_{k=1}^{n} a_j^T X_{tj}\right) \geq 0,$$

$$= \sum_{J=1}^{n}\sum_{k=1}^{n} a_j^T a_k\ E\{X_{tj} X_{tk}\} \geq 0,$$

$$= \sum_{J=1}^{n}\sum_{k=1}^{n} a_k^T \Gamma(t_j - t_k)\ a_j \geq 0 \quad \ldots.(6)$$

Hence proved.

Theorem 3.3

The auto covariance matrix of vector stationary time series is an even function of h. i.e.,

$\Gamma_{ij}(h) = \Gamma_{ij}(-h)^T$.

Proof:

Here, $\text{Cov}(X_i, Y_{i+1}) = \{\Sigma X_i Y_{i+1} - 1/n \Sigma X_i \Sigma Y_i\}/n$, If X_i, Y_i are different series.

$\text{Cov}(X_i, Y_{i+1}) \neq \text{Cov}(X_i, Y_{i-1})$

i.e. $\{\Sigma X_i Y_{i+1} - 1/n\Sigma X_i \Sigma Y_i\}/n \neq \{\Sigma X_i Y_{i-1} - 1/n\Sigma X_i \Sigma Y_{i+1}\}/n$

$\therefore X_1Y_2 \neq X_2Y_1$

When X_i, Y_i are identical series

$\Gamma(1) = \Gamma(-1)$, otherwise not true.

Hence, $\Gamma_{ij}(h) = \Gamma_{ij}(-h)^T$ proved.

Theorem 3.4

Let X_{it}'s be independently and identically distributed with $E(X_{it}) = \mu_i$ and $\text{var}(X_{it}) = \sigma_i^2$ then

$\Gamma_{ij}(t, k) = E(X_{it}, X_{jk}) = \sigma_i^2,\quad t = k = 0, t \neq k$

This process is stationary in the strict sense.

Testing Procedure

Using the model for table-5.1B

$X_i = (\beta_0)_i + (\beta_1)_i t_i + \in_i$, i =1, 2,……5 . . .. (7)

Where

i. X_i are annual male birth series X_i (t), i = 1, 2……5 for five districts.
ii. t_i are the time (in years) variable.
iii. $\in_i$, are a random error term normally distributed as mean zero and variance σ^2, i.e

$\in_i \sim N(0, \sigma^2)$.

Male birth X_i are the dependent variables and time t_i (in years) are independent variables.

Using the model for table-5.2A and 5.2B

$\Upsilon_{ij}(h) = (\beta_0)_{ij} + (\beta_1)_{ij}\, h + \in_{ij}$, i =1, 2, … 5 ; j = 1, 2, … 5 ...(8)

Where

i. $\Upsilon_{ij}(h)$ are auto-covariance values for individual series and auto-covariance matrices for vector time series .
ii. h are the lag values of variable.
iii. $\in_i$ are a random error term normally distributed as mean zero and variance σ^2, i.e. $\in_i \sim N(0, \sigma^2)$. Υ_{ij} (h) are the dependent variables and h are independent variables.

Υ_{ij}(h) are the dependent variables and h are independent variables.

Defining $\Upsilon_{ij}(h) = \text{cov}(X_i, X_{j+h})$, (i = 1, 2…5; j =1, 2…5) were computed for various values of h. Since the total series constituted of 31 values at least 10 values were included in the computation. The relation between Υ_{ij}(h) and h were examined the model in (table-5.2B).

Defining the $\Gamma_{ij}(h)) = \text{cov}(X_i, X_{j+h})$, covariance matrix with a stationary time series for observations $\bar{X} = (x_1, x_2, \ldots x_n)$ realizations and ρ_{ij} (h) = correlation (X_i, X_{j+h}) correlation matrix with a stationary time series for observations $\bar{X} = (x_1, x_2, \ldots x_n)$ realizations , 21 such matrices corresponding to h = 0 to 20 , define one series of matrices each 5×5 , and hence 25 component series were computed . The relation between Γ_{ij}(h) and h were examined the model in (table-5.3 and 5.4A) .

The method of testing intercept $(\beta_0)_{ij} = 0$ and regression coefficient $(\beta_1)_{ij} = 0$, [7]. Null hypothesis for test Statistic used to test and set up.

Inference Concerning Slope $(\beta_1)_{ij}$

For testing $H_0 : (\beta_1)_{ij} = 0$ Vs $H_1: (\beta_1)_{ij} > 0$ for α = 0.05 percent level using t distribution with degrees of freedom is equal to n – 2 were considered.

$t_{n-2} = b / s_b$

Where b is the slope of the regression line and $s_b = s_e / s_t$ and $s_e = [\,SSE / n - 2]^{1/2}$,

$SSE = (s_t^2 - s_{tx}^2 / s_t^2)$, $s_{tx} = \Sigma(t_i - \bar{t})(X_i - \bar{X})$

$s_t^2 = \Sigma(t_i - \bar{t})^2$; $s_x^2 = \Sigma(X_i - \bar{X})^2$.

The residual sum of squares or the sum of squares due to error is SSE.

The relation between Υ_{ij}(h) and h were examined the model in (table-5.2C and 5.5B)

From model, $\Upsilon_{ij}(h) = (\beta_0)_{ij} + (\beta_1)_{ij}\, h + \in_i$,

Where $\bar{\Upsilon} = [\Upsilon ij]$, $\bar{\beta}_0 = (\beta_0)_{ij}$, $\bar{\beta}_1 = (\beta_1)_{ij}$, and $\bar{\in} = \in_i$ are all 5×5 matrices.

Hence the above model can be re-written as,

$$\bar{\Upsilon} = \bar{\beta}_0 + \bar{\beta}_1 h + \bar{\in},$$

Table-5.5B was obtained by regressing values of Υ_{ij}(h) against h, by using this testing shows that , both the hypothesis $\bar{\beta}_0 = 0$ and $\bar{\beta}_1 = 0$, test is not positive. Table-5.3 formed the input for table-5.5 B. In other words, $\bar{\Upsilon}$ are not all zero, showing that X_{it}, $X_{i\,t+h}$ are not all independent and there is trend. Hence X_{it} are not strictly stationary.

Example of Vector Time Series: Regional Male and Female Data.

Here is a real instance of a vector time series. Male and female data of Marathawada region was obtained from five districts, namely ABD-Aurangabad, PBN-Parbhani, BED-Beed, OSM-Osmanabad, and NED-Nanded. The data were collected from **"**Maharastra Quarterly Bulletin of Economics and Statistics", Directorate of Economics and Statistics Govt. of Maharastra Bombay[2].

Hence, we have five dimensional time series t_i , i = 1, 2 , 3 , 4 , 5 corresponding to the districts Aurangabad , Parbhani , Beed , Osmanabad and Nanded respectively. Table 5.1, shows the results of descriptive statistics and Table 5.1B, shows linear trend analysis. All the linear trends were found to be not significant.

Over the years many scientists have analyzed rainfall, temperature, humidity, agricultural area, production and productivity of Marathwada region of Maharastra state [1,3,4,5,6,7,9,10,11,13,14] Most of them have treated the time series for each of the revenue districts as independent time series and tried to examine the stability or non-stability depending upon data series. Most of the times non-stability has been concluded and hence possibly any sort of different treatment was possibility never thought of. In this investigation we treat the series first and individual series and then as a vector time series shows that the vector time series are not stable.

Conclusion

For Individual Time Series

It was observed t values are therefore not significant for the 3 districts , except Parbhani district i.e. concluded that X_i does not depend on t for 3 districts [3]. Similarly, Υ_{ij}(h) does not depend on h in 4 districts to mean that, 'no linear relation' rather than 'no relation'. The testing shows that, for the hypothesis $(\beta_1)_i = 0$, test is positive for t in 3 and h in 4 districts about both male and female births table (5.1.2 and 5.1.3).

Generally it is expected male and female births (annual) over a long period at any region to be not stationary time series. These result does not conform with the series in Parbhani district.

For Vector Time Series

To conclude that a vector time series is stationary, it is necessary to test association between $\bar{\Upsilon}$ and h. The association between $\bar{\Upsilon}$ and h fails in Aurangabad /Aurangabad and Osmanabad / Aurangabad (shown in table-5.5B) it is concluded that a vector time series is not stationary [8].

Analysis of Regional Infrastructural Time Series

The same strategy of anlaysing first individual time series as scalar series and then treating the vector series as the regional time series has been adapted here for analyzing number of male, female birth series of five districts. Ultimately, this appears one of the properties of vector time series, discovered by Fuller (1976)[8]. If one of the component series is non stationary, the vector series as a whole is also non stationary.

It is noted that a situation where the four scalar series are having trends, however, one of the series Parbhani (table 5.2A) is stationary status when ARMA (Auto-Regressive Moving Average) is considered four series.

Scalar Time Series Analysis

To begin with, a straight forward analysis of the five series *of* number of male and female birth for districts has been done to test their trends. Linear regresion analysis of the data to determine its trend was carried out.

Table-5.1A

Elementary Analysis of Number of Male and Female Birth Time Series Data.

	Number of Male Birth					**Number of Female Birth**				
Dists	**ABD**	**PBN**	**OSM**	**BED**	**NED**	**ABD**	**PBN**	**OSM**	**BED**	**NED**
Mean	27024.0	14666.3	21213.5	11962.8	14843.1	23881.0	15428.8	17966.7	11411.6	10576.9
S.D.	8305.75	14858.0	7861.63	3939.46	11965.06	8148.6	19139.5	6133.1	6145.0	4312.2
C.V.	30.73	101.31	37.06	32.93	80.61	34.1	124.0	34.1	53.8	40.8

Table-5.1B

Linear Regression Analysis of Number of Male and Female Birth Data to Determine Trend.

		Number of Male Birth				**Number of Female Birth**			
Dists		**Coefficients**	**S. E.**	**t Stat**	**Sigfi**	**Coefficients**	**S. E.**	**t Stat**	**Sigfi**
ABD	β_0	13442.48	1225.40	10.97	S	11884.34	1694.17	7.01	S
	β_1	798.91	62.89	12.70*	S	705.68	86.95	8.12*	S
PBN	β_0	12171.66	5436.63	2.24	S	8706.66	6896.93	1.26	NS
	β_1	146.75	279.02	0.53	NS	395.42	353.96	1.12	NS
OSM	β_0	10824.52	1942.87	5.57	S	10040.73	1555.24	6.46	S
	β_1	611.12	99.71	6.13*	S	466.23	79.82	5.84*	S
BED	β_0	6735.66	968.73	6.95	S	6390.20	2008.03	3.18	S
	β_1	307.48	49.72	6.18*	S	295.38	103.05	2.87*	S
NED	β_0	8886.26	4223.12	2.10	S	8881.84	1546.00	5.75	S
	β_1	350.41	216.74	1.62	NS	99.71	79.34	1.26	NS

t =2.042 is the crical value for 31 d f at 5% L. S. *Shows the significant value.

The table 5.1A displays that in number of male and female birth series all of them have similar values of CV. This indicates that their dispersion is almost identical. In this relation, cognizable trend has been not identified in district Parbhani. In presence of linear trend, with reasonably low CV values can be taken as evidence of series being not stationary individually in rest of the four districts.

Further search for evidences of stability included determination of auto covariance and their dependency on lag variable h. Such an analysis requires an assumption of AR (Autoregrasive) model. Therefore a real test for stationary property of the time series can come by way of establishing auto covariances which do not depend on the lag variable.

Table-5.2A

Correlations between H and Auto Covariance are

	Number of Male Birth					**Number of Female Birth**				
Dists	ABD	PBN	OSM	BED	NED	ABD	PBN	OSM	BED	NED
r	**-0.984***	-0.349	**-0.896***	**-0.800***	**-0.456***	**-0.931***	-0.267	**-0.917***	**-0.702***	**-0.446***

Correlation coefficient r = 0.433 is the critical value for 19 df at 5% LS. *shows the significant value.

Correlations between $\gamma_{ij}(h)$ and h were found to be significant of number of male birth in 4 districts ABD, OSM, BED and NED showing that these 4 component time series can be reasonably assumed to be non stationary. The coefficient β_1 is significant, with negative value showing that all the 4 component series have been experiencing significantly declining number of male birth over the past years.

In case of number of female birth correlations between γ_{ij} (h) and h were found to be significant in 4 districts ABD, OSM, BED and NED showing that the time series can be reasonably assumed to be non stationary. The coefficient β_1 is significant, with negative value showing that all disticts have been experiencing significantly declining number of female birth over the past years.

Table-5.2B

Linear Regression Analysis of Lag Values vs Covariance.

		Number of Male Birth				Number of Female Birth			
Dists		**Coefficeints**	**S. E.**	**t Stat**	**Sigfi**	**Coefficients**	**S. E.**	**t Stat**	**Sigfi**
ABD	β_0	66467813.01	1783459.55	37.27	S	47755248.96	2498705.46	19.11	S
	β_1	-3727557.51	152556.94	-24.43*	S	-2367877.20	213739.00	-11.08*	S
PBN	β_0	30626576.22	20982617.04	1.46	NS	57073759.99	42676038.68	1.34	NS
	β_1	-2911059.62	1794850.80	-1.62	NS	-4406978.50	3650503.75	-1.21	NS
OSM	β_0	44300364.26	3514295.11	12.61	S	27760871.35	2054604.88	13.51	S
	β_1	-2643232.65	300612.43	-8.79*	S	-1762613.53	175750.68	-10.03*	S
BED	β_0	9312878.12	910997.79	10.22	S	14134178.74	2693028.57	5.25	S
	β_1	-452882.15	77926.65	-5.81*	S	-989981.75	230361.37	-4.30*	S
NED	β_0	31469011.01	13479568.80	2.33	S	4256205.34	2195753.78	1.94	NS
	β_1	-2578523.53	1153040.86	-2.24*	S	-407869.96	187824.54	-2.17*	S

t =2.093 is the crical value for 19 d f at 5% L. S. *Shows the significant value.

Table 5.3

Cov.(H, Γ_{IJ}(H)) Matrix Values About Number Male Birthand Female Birth Data

	Number of Male Birth					Number of Female Birth				
Dists	**ABD**	**PBN**	**OSM**	**BED**	**NED**	**ABD**	**PBN**	**OSM**	**BED**	**NED**
ABD	-136677108.6	-21658043.0	-113322648.5	-43074771.0	-73386029.7	-86822163.9	-57537978.4	-73793911.0	-59501344.3	-20286835.4
PBN	-46471732.3	-106738852.6	-55672400.8	-4850656.6	-76142414.9	-48580992.2	-161589211.6	-76423924.8	-41864835.1	-17962502.2
OSM	-109278518.6	24244835.5	-96918530.6	-38878061.7	-35037386.6	-74393767.3	-13310264.6	-64629162.7	-40810588.3	-3807650.6
BED	-48664241.7	-4679438.7	-40089066.7	-16605679.0	-27478033.9	-44558683.9	-26898463.3	-40185802.5	-36299330.7	-6998974.8
NED	-75212804.9	-12910537.3	-60463765.7	-14882488.0	-94545862.9	-12989056.8	-9018376.2	-14066171.4	-20535027.2	-14955232.0

Table 5.4A

$\rho_{ij}(h)$ = Correlation(h, $\Gamma_{ij}(h)$) Matrix Values about Number Male Birth and Female Birth Data

	Number of Male Birth					Number of Female Birth				
Dists	**ABD**	**PBN**	**OSM**	**BED**	**NED**	**ABD**	**PBN**	**OSM**	**BED**	**NED**
ABD	**-0.984***	-0.236	**-0.962***	**-0.957***	**-0.668***	**-0.931***	-0.422	**-0.945***	**-0.837***	**-0.484***
PBN	**-0.633***	-0.349	**-0.553***	-0.071	-0.266	**-0.506***	-0.267	**-0.711***	-0.321	-0.147
OSM	**-0.959***	0.309	**-0.896***	**-0.875***	-0.303	**-0.925***	-0.187	**-0.917***	**-0.749***	-0.133
BED	**-0.959***	-0.101	**-0.916***	**-0.800***	**-0.457***	**-0.908***	-0.230	**-0.878***	**-0.702***	-0.218
NED	**-0.885***	-0.150	**-0.750***	-0.398	**-0.456***	**-0.580***	-0.112	**-0.588***	**-0.663***	**-0.446***

Correlation coefficient r = 0.433 is the critical value for 19 d f at 5% L S. . *Shows the significant value.

Infrastructural Time Series of Five Districts Treated As A Single Vector Time Series

On treating the series together, one may look at them as a single vector time series representing the whole of the Marathwada region. We define vectors,

NMB = (nmb_1 , nmb_2 , nmb_3 , nmb_4 , nmb_5) having five components .

NFB = (nfb_1 , nfb_2 , nfb_3 , nfb_4 , nfb_5) having five components .

Here nmb_i and nfb_i are the number of male birth and female birth for the i^{th} district .

Auto variance and auto covariance matrices were computed for the vector time series (NMB, NFB).

Observe that,

A_1. The matrix $\Upsilon_{ij}(h)$ for h = 0 is symmetric , and for h > 0 they are all not symmetric. This is expected.

B_1. We have a series of 5×5 matrices $\Upsilon_{ij}(h)$, h = 0 ,1 , 2 , … 20 , and now onwords we are interested in behavior of this matrix series.

C_1. Out of the 25 components in number of male birth series 16 series showed significant (coefficients) intercepts and slope. Due to the significant values in Table 6.3.7 the districts ABD-0.984* , OSM-0.896*, BED-0.800*, NED-0.456* individually and ABD/OSM-0.962*, ABD/BED-0.957*, ABD/NED-0.668*, PBN/ABD-0.633*, PBN/OSM-0.553*, OSM/ABD-0.959* , OSM/BED-0.875*, BED/ABD-0.959*, BED/OSM-0.916*, BED/NED-0.457*, NED/ABD-0.885* , NED/OSM-0.750* in combinations seem to be causing the variations responsible for the non-stationary nature of the series . That is both hypothesis $\beta_0 = 0$ and $\beta_1 = 0$ could be rejected. Which means the model Eq(8) (matrix equation in 5×5 matrices) with hypothesis $\beta_0 = 0$ and $\beta_1 = 0$ was not validated.

Out of the 25 components in number of female birth series 16 series showed significant (coefficients) intercepts and slope. Due to the significant values in Table 6.3.7 the districts ABD-0.931* , OSM-0.917*, BED-0.749*, NED-0.446* individually and ABD/OSM-0.945*, ABD/BED-0.837*, ABD/NED-0.484*, PBN/ABD-0.506*, PBN/OSM-0.711*, OSM/ABD-0.925* , OSM/BED-0.749*, BED/ABD-0.908*, BED/OSM-0.878*, NED/ABD-0.580* , NED/OSM-0.588*, NED/BED-0.663* in combinations seem to be causing the variations responsible for the nonstationary nature of the series.

Hence we can consider the vector time series to be not stationary. Thus we have a situation where when treated as individual series 4 components in number of male birth as well as in number of female birth are not stationary and when treated as vector time series the whole of the vector time series is not stationary.

When we think of the number male and female birth of whole of the region individually and in combinations seem to be causing the variations responsible for the nonstationary nature of the series. Hence we may conclude that the regional number male and female births are not stationary and must have a trend.

Regional View of the Infrastructural Aspects

A regional view of the infrastructural aspects helps us in classifying the region into subgroups of similar infrastructural characteristics. From results it is clear that PBN is stationary in number of male birth as well as female birth time series. When the vector time series is considered the region showed non-stationary behavior.

Summary of the Number of Male Birth and Female Birth Results

S. No.	Factors	Scalar time series		Vector Time Series
		Stationary	Not Stationary	
1	No. of Male Birth	PBN	ABD, OSM, BED. NED	Not stationary at $\rho_{11}(h)$, $\rho_{13}(h)$ $\rho_{14}(h)$, $\rho_{15}(h)$, $\rho_{21}(h)$, $\rho_{23}(h)$, $\rho_{31}(h)$, $\rho_{33}(h)$, ρ_{34} (h), $\rho_{41}(h)$, $\rho_{43}(h)$, $\rho_{44}(h)$, $\rho_{45}(h)$ $\rho_{51}(h)$, $\rho_{53}(h)$, $\rho_{55}(h)$
2	No. of Female Birth	PBN	ABD, OSM, BED NED	Not stationary at $\rho_{11}(h)$, $\rho_{13}(h)$ $\rho_{14}(h)$, ρ_{15} (h) , $\rho_{21}(h)$,$\rho_{23}(h)$, $\rho_{31}(h)$, $\rho_{33}(h)$, ρ_{34} (h), $\rho_{41}(h)$, $\rho_{43}(h)$, $\rho_{44}(h)$, $\rho_{51}(h)$ $\rho_{53}(h)$, $\rho_{54}(h)$, $\rho_{55}(h)$

1. Time series for ABD, OSM , BED and NED has been unstable in number of male birth, female birth.
2. Time series for PBN has been stable in number of male and female birth.

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Conclusion

For individual time series

It was observed t values are therefore not significant for the 3 districts , except Parbhani district i.e. concluded that X_i does not depend on t for 3 districts [3]. Similarly, $\Upsilon_{ij}(h)$ does not depend on h in 4 districts to mean that, 'no linear relation' rather than 'no relation'. The testing shows that, for the hypothesis $(\beta_1)_i = 0$, test is positive for t in 3 and h in 4 districts about both male and female births table (5.1.2 and 5.1.3).

Generally it is expected male and female births (annual) over a long period at any region to be not stationary time series.These result does not conform with the series in Parbhani district.

a) For Vector Time Series

To conclude that a vector time series is stationary, it is necessary to test association between $\bar{\Upsilon}$ and h. The association between $\bar{\Upsilon}$ and h fails in Aurangabad /Aurangabad and Osmanabad / Aurangabad (shown in table-5.5B) it is concluded that a vector time series is not stationary [8].

b) Analysis of Regional Infrastructural Time Series

The same strategy of anlaysing first individual time series as scalar series and then treating the vector series as the regional time series has been adapted here for analyzing number of male, female birth series of five districts. Ultimately, this appears one of the properties of vector time series, discovered by FULLER (*1976)*[8]. If one of the component series is non stationary, the vector series as a whole is also non stationary.

It is noted that a situation where the four scalar series are having trends, however, one of the series Parbhani (table 5.2A) is stationary status when ARMA (Auto-Regressive Moving Average) is considered four series.

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